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ON SYMMETRIC REINFORCEMENTS OF CIRCULAR CUTOUTS

by

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ON SYMMETRIC REINFORCEMENTS OF CIRCULAR CUTOUTS[1]

by

E. Levin[2]

A slab with a circular cutout is subjected to stresses in the plane of the slab. The cutout has removed material which would have participated in carrying the load, hence the slab with cutout will fail under the application of stresses which the complete slab could have supported. In order to eliminate at least part of this weakening, the slab with cutout may be reinforced by the addition of material about the cutout. Such a reinforcement may, of course, be designed in any shape. The present paper is concerned with extending the results of Weiss, Prager, and Hodge [1][3] for a cylindrical reinforcement to a reinforcement of arbitrary symmetric cross-section. In particular, a method will be described for the determination of a lower bound on the collapse load for a slab with circular cutout and a general symmetric reinforcement.

1. Introduction.

Consider a uniform plane rectangular slab with a centered circular cutout of radius a. The cutout is reinforced by additional material extending to the radius $a + \delta$; the cross-section of the reinforcement is prescribed (Fig. 1). The slab is subjected

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2. Graduate Student, University of California, Los Angeles.

3. Numbers in square brackets refer to the Bibliography at the end of the paper.

to arbitrary uniform tensions along its outer edges and the inner edge is stress free.

The present paper is concerned with the determination of a lower bound on those values of the loads which may be considered safe. In the next section, the physical assumptions made in discussing the problem will be stated, and a precise mathematical problem formulated. In particular, a domain of safe loads will be defined. Section 3 is concerned with the problem of equal biaxial tensions. The next three sections are concerned with a reinforcement whose cross-section is symmetric about both axes through its center and whose boundary in the first quadrant is monotonically non-increasing.* Some general results are established for all symmetric reinforcements by approximating the true interaction curve by a parabola. It is shown that the cylindrical reinforcement, which was first solved by Weiss, Prager, and Hodge [1], appears as a special case. An estimate of the error introduced by the parabolic approximation is obtained for a quasi-toroidal reinforcement. Limitations and conclusions concerning the results are presented in section 7.

2. Statement of the Problem.

The analysis will be based on a limit theorem of Drucker, Greenberg, and Prager [2] which says that a given system of loads is safe if stresses can be found which

1) are in equilibrium with the given loads, and
2) do nowhere exceed the limits set by the yield criterion.

* Non-symmetric reinforcements may be treated numerically. The method is illustrated with a bevelled reinforcement in [5].

The slab and the reinforcement are assumed to be of a uniform perfectly plastic material which satisfies Tresca's yield criterion [3].

Following Weiss, Prager, and Hodge [1], the reinforced slab will be regarded as consisting of a uniform part under conditions of plane stress, and a hub which behaves as a curved beam (Fig. 1c). Let the applied tractions on the outer edge of the slab be $\lambda_x s$ and $\lambda_y s$, where s is the yield strength of the material in simple tension and it is assumed that the applied loads would be safe if there were no cutout. It follows that in the uniform part of the slab, the homogeneous state of stress $\sigma_x = s\lambda_x$, $\sigma_y = s\lambda_y$, $\tau_{xy} = 0$ satisfies the two conditions of the Drucker, Greenberg, Prager theorem, and transmits total loads of intensities $hs\lambda_x$, $hs\lambda_y$ per unit length to the hub. The problem, then, is to find values of λ_x, λ_y which lead to a safe stress distribution in the hub.

It has previously been shown [1, 4] that the safe states of loading for the uniform slab without cutout may be represented by the points on and inside a hexagon, and that any such safe state can be obtained from the stress-free state, the state of uniaxial tension and the state of biaxial tension, by the following operations:

(a) If T_x, T_y is a safe load, then $-T_x$, $-T_y$ is also a safe load.

(b) If T_x', T_y' and T_x'', T_y'' are each safe loads, then $\mu T_x' + (1-\mu)T_x''$, $\mu T_y' + (1-\mu)T_y''$ is a safe load for any $0 \leq \mu \leq 1$.

For the reinforced cut slab, the <u>domain of safe loads</u> will be

defined as the largest hexagon which may be obtained from the stress-free, uniaxial, and biaxial states by these operations. Hence the problem of determining the domain of safe loads is reduced to the problems of finding lower bounds for λ in biaxial tension ($\lambda_x = \lambda_y = \lambda$) and uniaxial tension ($\lambda_x = \lambda$, $\lambda_y = 0$). The case of biaxial tension is fairly simple and may be treated for all reinforcements simultaneously. Uniaxial tension is more difficult and only symmetric reinforcements will be considered.

For a curved beam the state of stress in any section is specified by a bending moment M and an axial force N caused by tensile stresses, and a shear force caused by shear stresses. The effect of the shear force is negligible for solid cross sections and will be disregarded in the subsequent analysis.

3. Biaxial Tension.

Consider a cutout slab shown in Fig. 1, and let A be the total cross-sectional area of the hub. In biaxial tension the hub is subjected to a uniformly distributed radial force of magnitude $h\lambda s$ per unit length of the circumference of radius $(a + \delta)$. To satisfy the equilibrium requirement, the stress resultants in the hub must be given by

$$N = (a + \delta)h\lambda s, \quad M = 0 \tag{3.1}$$

at all cross sections. Such resultants are furnished by a uniform tensile stress of magnitude N/A. From the yield criterion this stress cannot exceed the yield stress s, hence the axial force must satisfy

$$N \leq As. \tag{3.2}$$

Combining (3.1) and (3.2) leads to the inequality

$$\lambda \leq \lambda_1 \equiv A/[h(a + \delta)]. \tag{3.3}$$

Hence for the case of biaxial tension any value of λ satisfying Inequality (3.3) furnishes a lower bound.

4. General Symmetric Reinforcement.

The type of reinforcement considered here has the following properties:

1) the hub is symmetric about the x and z axes, where the x axis lies in the plane of the slab and passes through the center of mass of the hub and through the center of the cutout, and the z axis is normal to the slab and also passes through the center of mass of the hub, and

2) in the first quadrant the boundary of the hub, $z = f(x)$, is a continuous, single-valued, monotonically non-increasing function of x (Fig. 2a).

To find limiting combinations of bending moment M and axial force N which the beam can support, i.e., which do not violate the yield criterion, consider a fully plastic stress distribution of the type shown in Fig. 2b. Part of the section is stressed to the yield limit in tension and the rest in compression. The absolute values of the bending moment and axial force for such a stress distribution are

$$|N| = |s(2\int_{\zeta}^{\delta/2} f(x)dx - \int_{-\delta/2}^{\zeta} 2\, f(x)dx)| = |4s\int_{0}^{\zeta} f(x)dx, \tag{4.1}$$

$$|M| = |s(2\int_{\zeta}^{\delta/2}\int_{0}^{f(x)} xdxdz - 2\int_{-\delta/2}^{\zeta}\int_{0}^{f(x)} xdxdz)| = |4s\int_{\zeta}^{\delta/2} x\, f(x)dx|. \tag{4.2}$$

If N/s and M/s are regarded as Cartesian co-ordinates in a stress-resultant space, then (4.1) and (4.2) define an interaction curve in the NM plane. It can be shown that this curve will be closed and will bound a convex, symmetric region which will be defined as the <u>safe region</u>. Any point within the safe region represents a bending moment and an axial force which do not violate the yield criterion. The boundary of the region is described parametrically in terms of ζ and represents the limiting admissible combinations of bending moment and axial force.

The exact safe region is defined by Eqs. (4.1) and (4.2), which contain integrals of the arbitrary function f(x) and xf(x). Therefore, it is not possible to obtain a direct limiting relation between N and M. However, any region which lies wholly within the safe region will itself consist only of safe stress resultants. It will be shown that the domain bounded in each quadrant by a certain parabolic arc is such a safe approximation.

From symmetry, it is only necessary to consider the first quadrant. Here

$$M/s = 4 \int_{\zeta}^{\delta/2} xf(x)dx, \tag{4.3}$$

$$N/s = 4 \int_{0}^{\zeta} f(x)dx. \tag{4.4}$$

Define

$$M_0/s \equiv 4 \int_{0}^{\delta/2} xf(x)dx, \tag{4.5}$$

$$N_0/s \equiv 4 \int_{0}^{\delta/2} f(x)dx, \tag{4.6}$$

where M_o and N_o represent the extreme values in the positive M and N directions respectively. Thus the end points of the interaction curve in the first quadrant are $(0, M_o/s)$ and $(N_o/s, 0)$ which correspond to $\zeta = 0$ and $\zeta = \delta/2$ respectively. The approximating curve will be taken as the parabola

$$M/M_o = 1 - (N/N_o)^2 \tag{4.7}$$

which passes through the same endpoints with a horizontal tangent at $N = 0$.

In order to prove that this curve is an admissible approximation it is sufficient to show that for any given value of ζ between zero and $\delta/2$, the ordinate M/M_o of the approximate curve never exceeds the ordinate M/M_o of the interaction curve. In terms of ζ, the ordinate of the approximate curve is

$$M/M_o = 1 - (N/N_o)^2 = 1 - \left[\frac{\int_0^{\zeta} f(x)dx}{\int_0^{\delta/2} f(x)dx}\right]^2, \tag{4.8}$$

while the ordinate of the interaction curve is

$$M/M_o = \frac{\int_{\zeta}^{\delta/2} xf(x)dx}{\int_0^{\delta/2} xf(x)dx}. \tag{4.9}$$

Therefore for all $0 \leq \zeta \leq \delta/2$, it must be shown that

$$g(\zeta) \equiv \frac{\int_{\zeta}^{\delta/2} xf(x)dx}{\int_0^{\delta/2} xf(x)dx} - 1 + \left[\frac{\int_0^{\zeta} f(x)dx}{\int_0^{\delta/2} f(x)dx}\right]^2 \geq 0. \tag{4.10}$$

It is evident from the definition of the approximate

curve that $g(0) = g(\frac{\delta}{2}) = 0$, and $g'(0) = 0$. In the appendix it is shown that $g'(\frac{\delta}{2})$ is equal to or less than zero, and that the equation $g'(\zeta) = 0$ has not more than two roots in the closed interval $0 \leq \zeta \leq \delta/2$. Since $g(\zeta)$ is continuously differentiable, it follows that in addition to its extremum at $\zeta = 0$, it has only one relative extremum between zero and $\delta/2$. Since the slope is non-positive at $\zeta = \delta/2$, this extremum must be a maximum. Therefore, Inequality (4.10) is valid, since otherwise $g(\zeta)$ would have a minimum in the interior.

Thus the approximation is safe and the yield condition for the hub may be approximated by

$$|M/M_o| \leq 1 - (N/N_o)^2. \tag{4.11}$$

Under uniaxial tension (Fig. 3) the stress resultants M and N at any cross section Θ of the hub may be computed from equilibrium considerations to within a redundant bending moment, X. This statically indeterminate quantity represents the bending moment at the cross section $\Theta = 0$ and from equilibrium is seen to be the only non-vanishing stress resultant there. The stress resultants are found to be

$$N = F \sin\Theta = h\lambda s(a+\delta)\sin^2\Theta, \tag{4.12}$$

$$M = X + \int_0^{(a+\delta)\sin\Theta} h\lambda s\left[(a+\delta/2)\sin\Theta - y\right]dy = X + \frac{1}{2}h\lambda s a(a+\delta)\sin^2\Theta, \tag{4.13}$$

or

$$N/N_o = d\sin^2\Theta, \tag{4.14}$$

$$M/M_o = Y + c\sin^2\Theta, \tag{4.15}$$

where $Y = X/M_o$, $c(\lambda) = h\lambda sa(a+\delta)/2M_o > 0$, $d(\lambda) = h\lambda s(a+\delta)/N_o > 0$. The substitution of Eqs. (4.14), (4.15) into the yield condition (4.11) leads to

$$Y + c\sin^2\theta \leq 1 - d^2\sin^4\theta, \tag{4.16a}$$

$$-Y - c\sin^2\theta \leq 1 - d^2\sin^4\theta. \tag{4.16b}$$

These inequalities must be satisfied for all $0 \leq \theta \leq \pi/2$.

Let $w = \sin^2\theta$. Then (4.16) may be written

$$P(w) = d^2w^2 + cw + Y - 1 \leq 0, \tag{4.17a}$$

$$Q(w) = d^2w^2 - cw - Y - 1 \leq 0. \tag{4.17b}$$

Here P and Q represent parabolas concave upwards, hence a necessary and sufficient condition that Inequalities (4.17) hold for all $0 \leq w \leq 1$, is that they hold at both end points. Thus

$$-1 \leq Y \tag{4.18}$$

$$d^2 - c - 1 \leq Y \tag{4.19}$$

$$Y \leq 1 \tag{4.20}$$

$$Y \leq 1 - c - d^2. \tag{4.21}$$

The actual choice of Y is immaterial, provided only that some Y exists which satisfies all of the above inequalities. A necessary and sufficient condition that this be the case is that each left hand sido be less than each right hand side. Thus

$$- 1 \leq 1, \tag{4.22}$$

$$- 1 \leq 1 - c - d^2, \tag{4.23}$$

$$d^2 - c - 1 \leq 1 - c - d^2, \tag{4.24}$$

$$d^2 - c - 1 \leq 1. \tag{4.25}$$

Since c and d are positive, the above inequalities are equivalent to

$$d^2 + c \leq 2, \tag{4.26}$$

$$d \leq 1. \tag{4.27}$$

In view of the definition of d, (4.27) leads to

$$\lambda \leq 4 \int_0^{\delta/2} f(x)dx/h(a + \delta) \equiv \lambda_1 \tag{4.28}$$

where λ_1 is defined in Eq. (3.3).

With the aid of the definitions of d, c, and λ_1, together with $D = 4M_o/N_o$, (4.26) may be written

$$F(\lambda) = \lambda^2 + (2a/D)\lambda_1\lambda - 2\lambda_1^2 \leq 0. \tag{4.29}$$

$F(\lambda)$ represents a parabola concave upwards, hence (4.29) will be satisfied for all λ between the two roots of $F(\lambda) = 0$. Since the smaller root is negative and λ is positive, the restriction on λ becomes

$$\lambda \leq \lambda_2 \equiv \lambda_1[\sqrt{2 + (a/D)^2} - (a/D)]. \tag{4.30}$$

Thus any value of λ satisfying both (4.28) and (4.30) furnishes a lower bound for uniaxial loading. It follows from (4.28) and (3.3) that if λ is safe for uniaxial loading it is

also safe for biaxial loading. Therefore, the domain of safe loads is fully defined by

$$\lambda \leq \min(\lambda_1, \lambda_2), \tag{4.31}$$

where λ_1 is given by (3.3) and λ_2 by (4.30). In particular it may readily be shown that (4.31) is equivalent to

$$\left.\begin{aligned} \lambda &\leq \lambda_1 \quad \text{if} \quad a/D \leq \tfrac{1}{2}, \\ \lambda &\leq \lambda_2 \quad \text{if} \quad a/D \geq \tfrac{1}{2}. \end{aligned}\right\} \tag{4.32}$$

5. Cylindrical Ring Reinforcement.

The results of the preceding section are readily applicable to the case of a reinforcement consisting of a hollow cylindrical ring about the cutout (Fig. 4). The hub cross section is now a rectangle and $z = f(x)$ is a constant, $H/2$, where H represents the total height of the reinforcement. Hence

$$\lambda_1 = \frac{H\delta}{h(a + \delta)}, \tag{5.1}$$

$$D = \delta, \tag{5.2}$$

$$\lambda \leq \frac{H\delta}{h(a + \delta)} \quad \text{if} \quad \frac{a}{\delta} \leq \frac{1}{2}, \tag{5.3}$$

$$\lambda \leq \frac{H}{h} \left(\frac{1}{1 + (\delta/a)}\right) \left\{ \sqrt{1 + 2(\delta/a)^2} - 1 \right\} \text{ if } \frac{a}{\delta} \geq \frac{1}{2}. \tag{5.4}$$

For a "full strength" reinforcement of this type, $\lambda = 1$, and the ring must be so designed that

$$H/h = \frac{1 + (\delta/a)}{(\delta/a)} \text{ if } (a/\delta) \leq \frac{1}{2}, \tag{5.5}$$

$$H/h = \frac{1 + (\delta/a)}{\sqrt{1 + 2(\delta/a)^2} - 1} \text{ if } \frac{a}{\delta} \geq \frac{1}{2}. \tag{5.6}$$

These results coincide with those obtained directly by Weiss, Prager, and Hodge [1]. The exact agreement is accounted for by the fact that for this case only the actual safe region in the stress resultant plane is bounded by an interacting curve which consists of parabolic arcs. Hence the parabolic approximation used in section 4, coincides with the true interaction curve.

6. Quasi-toroidal Reinforcement.

Consider a reinforcement of the type shown in Fig. 5 where the boundary of the hub cross section is an arc of a circle, $f(x) = \sqrt{r^2 - x^2}$. A lower bound may readily be obtained from Inequalities (4.28) and (4.30). Further, an upper bound may be obtained for the error introduced by approximating the true interaction curve by a parabolic arc.

It can be shown that the interaction curve in the first quadrant is given by

$$\frac{1}{2} u = \cos^{-1}(v_1)^{1/3} + (v_1)^{1/3} \sqrt{1 - (v_1)^{2/3}}, \qquad (6.1)$$

where $u = N/sr^2$

and $v_1 = \frac{3}{4} M/sr^3 + (h/2r)^3.$

The appropriate parabolic approximation is

$$(v/v_o) = 1 - (u/u_o)^2 \qquad (6.2)$$

where

$$v = M/sr^3 = \frac{4}{3}[v_1 - (h/2r)^3],$$

$$v_o = M_o/sr^3 = \frac{4}{3}[1 - (h/2r)^3],$$

$$u_o = N_o/sr^2 = 2[\cos^{-1}(h/2r) + (h/2r)\sqrt{1 - (h/2r)^2}].$$

The curve (6.1) may be plotted in a uv_1 space. However, the parabolic approximation as well as the actual position of the u axis depends upon the ratio h/2r. To obtain an estimate of the error made by using the parabolic approximation (6.2), a second parabola is passed through the points ($u = 0$, $v_1 = 1$), ($u = \pi$, $v_1 = 0$). Such a parabola is

$$v_1 = 1 - (u/\pi)^2. \tag{6.3}$$

It can be shown by methods similar to those employed in section 4 that the limiting parabola (6.3) lies below the approximating parabola (6.2) for all values of h/2r, so that, independently of the dimensions of the reinforcement, the approximating parabola must lie between the limiting parabola and the interaction curve. As may be seen from Fig. 6 or Table I the approximation is a close one. The dotted curve represents a typical approximating parabola for the case h/2r = .5.

At least for this type of reinforcement (and of course for the cylindrical rings of section 5) the approximation appears to be very satisfactory. In view of the rather drastic physical approximations which were introduced in setting up the problem in section 2, it appears entirely reasonable to adopt the parabolic approximation to the yield condition.

7. Conclusions and Limitations.

The results in section 4 for a symmetric reinforcement provide methods for obtaining an approximate lower bound on the collapse load. The principal approximation made was that of treating the hub as a curved beam with no shear. Although this

has proved successful in the elastic range [6] certain limitations must be observed when applying the results. If the maximum thickness of the hub is much larger than that of the slab, the question of the carrying capacity arises. If on the other hand, the width becomes too large in comparison with the inner radius, the entire concept of a curved beam becomes questionable. Buckling in compression is not considered and must be treated separately.

The above approximations tend to predict an optimistic load capacity. On the other hand, the assumption of perfect plasticity which neglects strain-hardening will make the estimate conservative. Also, the slab was regarded as being in a homogeneous state of stress and no stress concentrations in the slab were considered.

At present there is insufficient experimental evidence available to determine the extent to which these various influences tend to cancel.

Appendix.

The differentiation of Eq. (4.10) with respect to ζ leads to

$$g'(\zeta) = \frac{-\zeta f(\zeta)}{\int_0^{\delta/2} x f(x)dx} + 2\left[\frac{\int_0^{\zeta} f(x)dx}{\left(\int_0^{\delta/2} f(x)dx\right)^2}\right] f(\zeta) \qquad \text{(A1)}$$

and

$$g'(\tfrac{\delta}{2}) = \frac{f(\frac{\delta}{2})32s^2}{M_0 N_0}\left[\int_0^{\delta/2} x f(x)dx - \frac{\delta}{4}\int_0^{\delta/2} f(x)dx\right]. \qquad \text{(A2)}$$

The sign of $g'(\frac{\delta}{2})$ will be the same as the sign of the bracketed

expression since the factor outside the brackets is always positive. By combining the two integrals and performing a change of variable, $t = (x - \delta/4)$, it is found that

$$g'(\delta/2) = \frac{f(\delta/2)32s^2}{M_0\ N_0} \left[\int_0^{\delta/4} t \left\{ f(\delta/4 + t) - f(\delta/4 - t) \right\} dt. \right. \qquad (A3)$$

But throughout the range of integration $f(\delta/4 + t) \leq f(\delta/4 - t)$ since f is monotonically non-increasing. Hence the integrand is non-positive throughout the range of integration and

$$g'(\delta/2) \leq 0. \qquad (A4)$$

In fact $g'(\delta/2) = 0$ can hold if and only if $f(\delta/4 + t) = f(\delta/4 - t)$ for all $0 \leq t \leq \delta/4$, i.e., $f(\zeta) = c$ and the hub is a rectangle. For this case the interaction curve is itself parabolic and $g(\zeta) \equiv 0$.

Consider the number of solutions of $g'(\zeta) = 0$. Since $f(\zeta) \neq 0$, the problem is to determine the number of solutions of

$$h(\zeta) \equiv K\zeta - \int_0^{\zeta} f(x)dx = 0, \qquad (A5)$$

where $K = \left[\int_0^{\delta/2} f(x)dx\right]^2 / 2\left[\int_0^{\delta/2} xf(x)dx\right]$. The number of distinct zeros of $h(\zeta)$ may exceed by one the number of sign changes of its derivative

$$h'(\zeta) = K - f(\zeta). \qquad (A6)$$

Since f is monotonically non-increasing and K is constant, $h'(\zeta)$ may change sign only once. Therefore $h(\zeta)$ and hence $g'(\zeta)$ have at most two distinct zeros. But $g'(0) = 0$, thus there is only one extremum in the interval which must represent a maximum since $g'(\delta/2) < 0$ whenever $g(\zeta) \not\equiv 0$.

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Table I

v_1	u, interaction	u, typical parabola	u, limiting parabola
1.00	0.0000	0.0000	0.0000
0.95	0.7287	0.7077	0.7025
0.90	1.0299	1.0009	0.9935
0.85	1.2595	1.2257	1.2167
0.80	1.4521	1.4154	1.4050
0.75	1.6211	1.5824	1.5708
0.70	1.7730	1.7334	1.7207
0.65	1.9118	1.8724	1.8586
0.60	2.0406	2.0016	1.9369
0.55	2.1606	2.1230	2.1074
0.50	2.2735	2.2378	2.2214
0.45	2.3801	2.3471	2.3299
0.40	2.4812	2.4515	2.4335
0.35	2.5775	2.5515	2.5328
0.30	2.6693	2.6480	2.6285
0.25	2.7571	2.7408	2.7207
0.20	2.8411	2.8307	2.8099
0.15	2.9215	2.9178	2.8964
0.10	2.9985	-----	2.9804
0.05	3.0720	-----	3.0621
0.00	3.1416	-----	3.1416

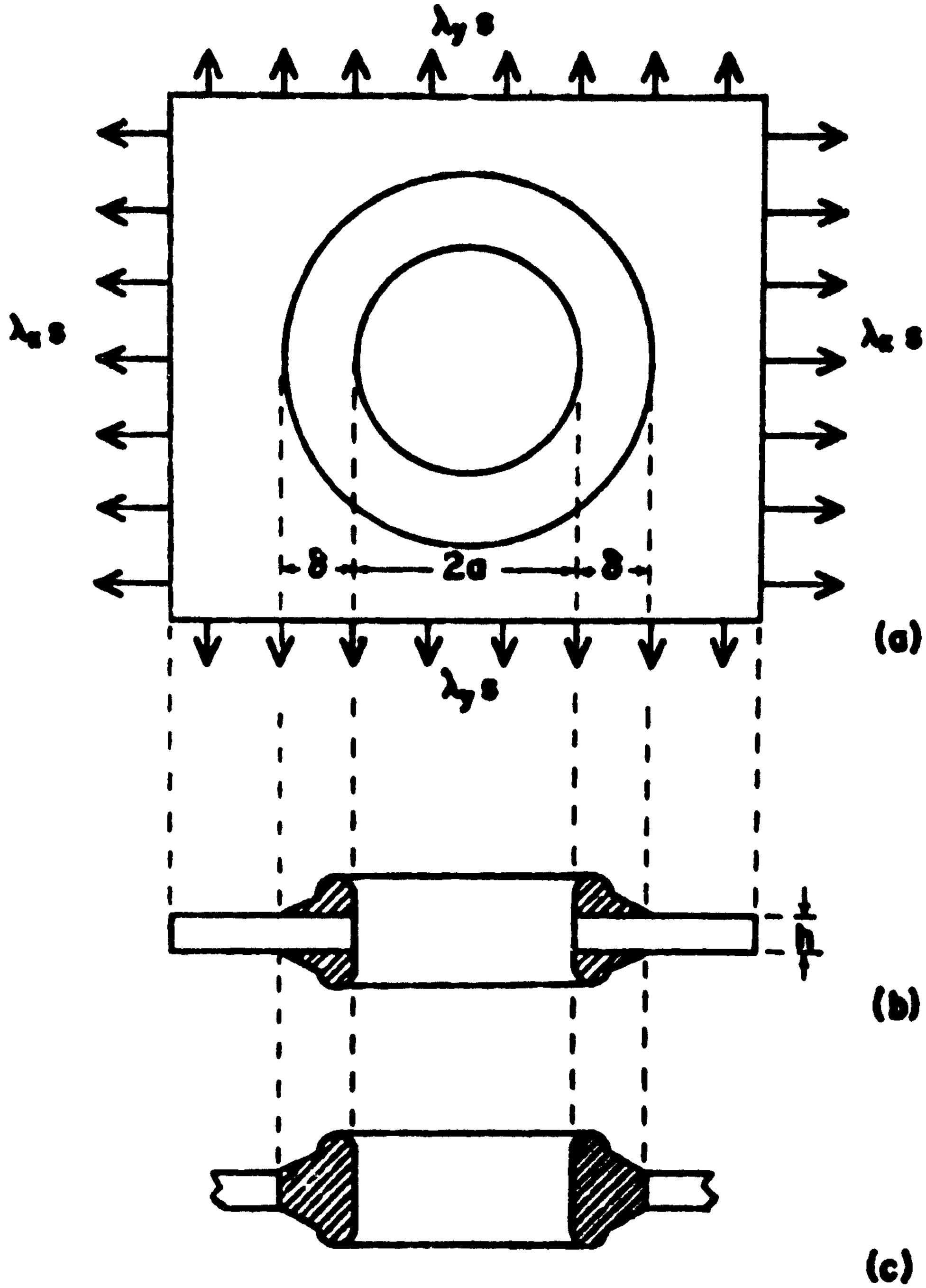

Fig. 1. Plane Slab with Reinforced Cutout.
(a) Plan View;
(b) Transverse Section;
(c) "Hub" and Slab.

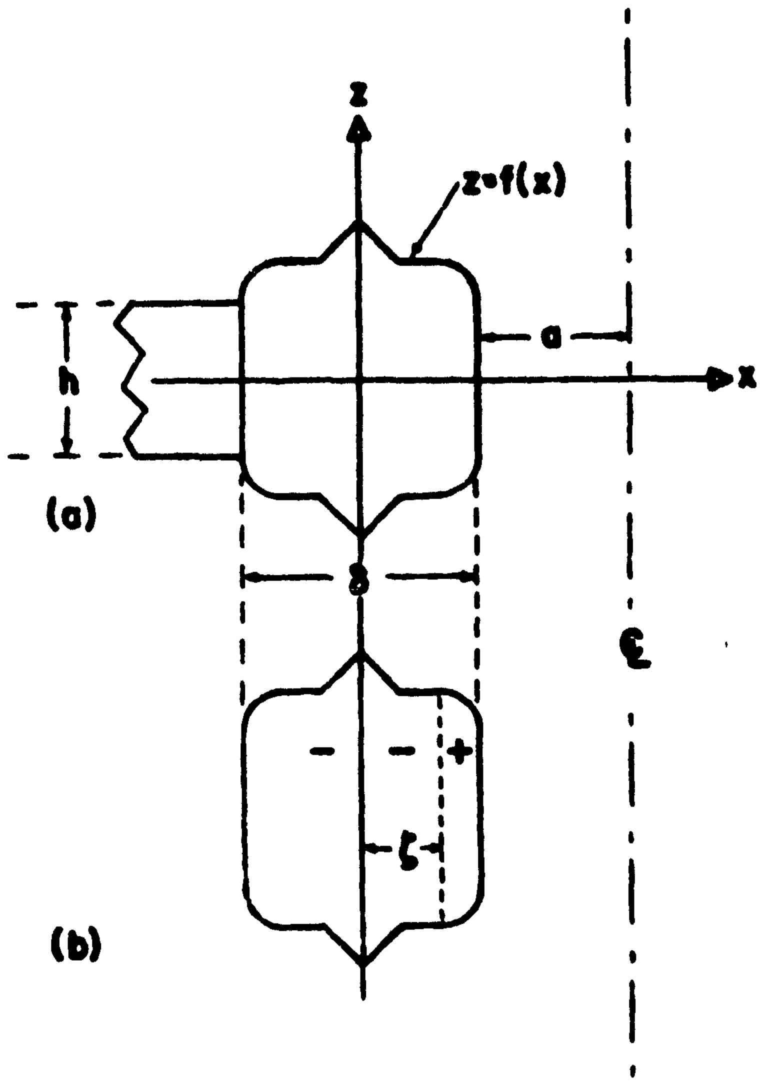

Fig. 2. (a) General Symmetric Hub;
(b) Fully Plastic Stress Distribution.

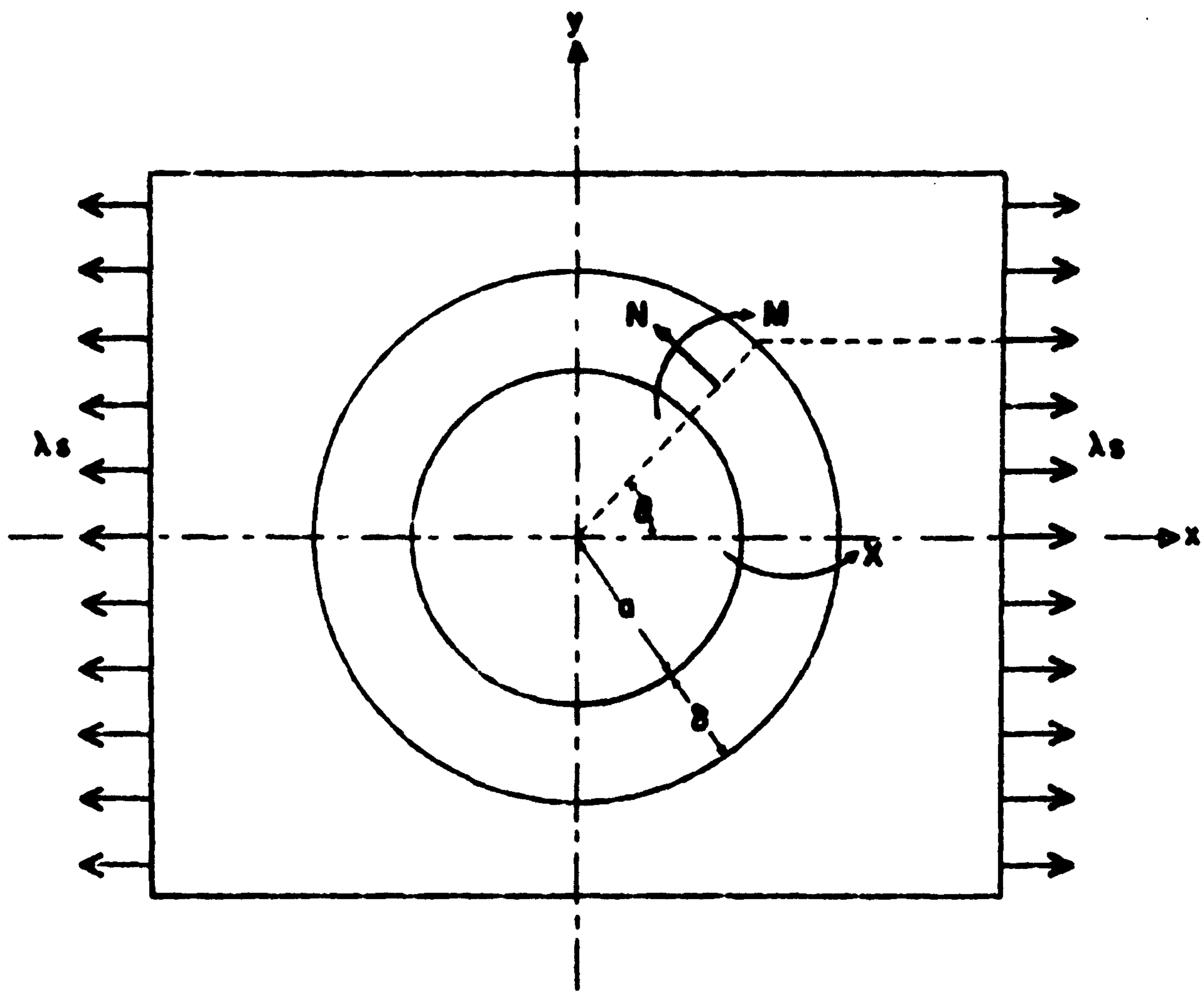

Fig. 3. Uniaxial Tension

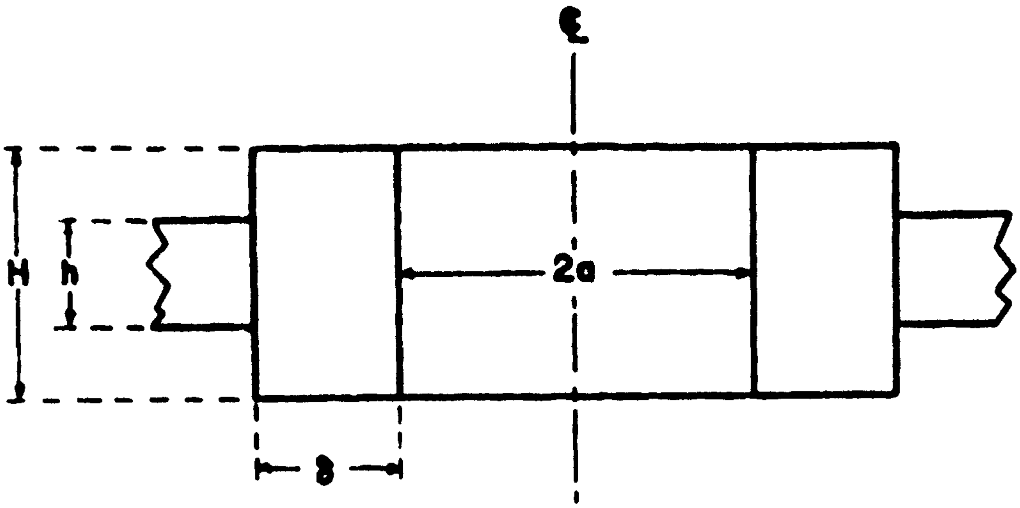

Fig. 4. Cylindrical Reinforcement

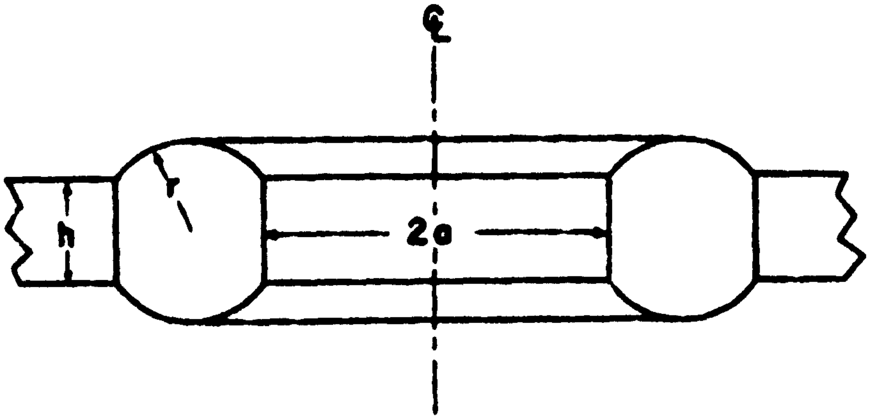

Fig. 5. Quasi-toroidal Reinforcement

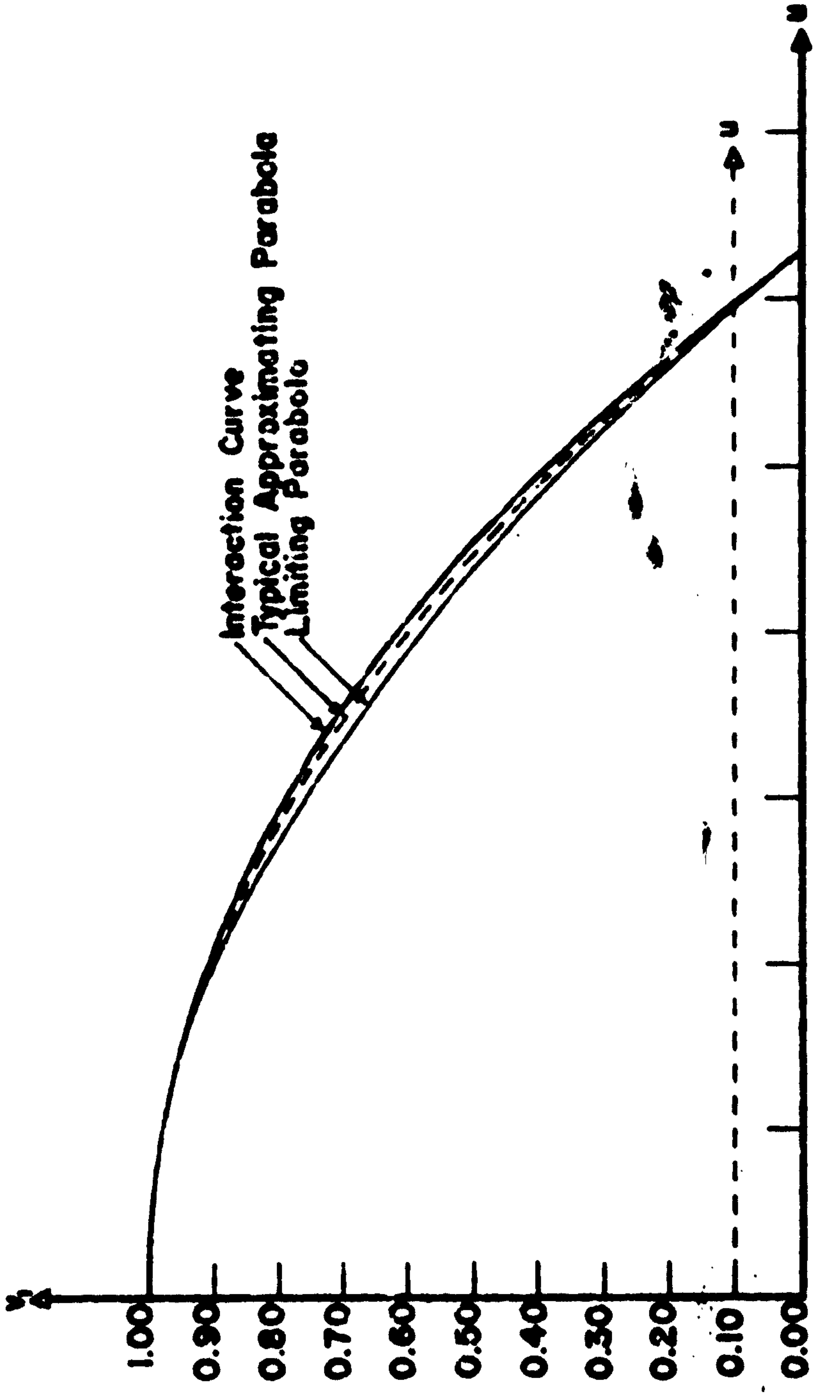

Fig. 6. Interaction Curve and Approximating Parabolas

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